Combinatorial Auctions

Multiplicative Prices Update

Online Combinatorial Auctions

- Bidders arrive sequentially.
- The mechanism decides which bundle she gets and at which price before it moves to the next bidder.

High Level Idea (Multiplicative Price Update):

1. Announce item prices to the bidder
2. Let her choose (demand oracle) the bundle she wants most
3. Update prices

Randomized Rounding on the fly [KV12]

- What if we could just use the bidders' valuations to learn the "right" prices to sell each item?
- Consider an algorithm that is allowed to sell more copies of each item than there are available.

1. Initialize parameter $P_0 = \frac{L}{4km}$.
2. For each bidder $i \in [n]$:
   - Ask the demand oracle of the bidder to choose a bundle $S_i \subseteq$ U.
   - Update $P_{i+1}^{j} = P_i^{j} \cdot 2^{1/k}$ for each item $j \in S_i$.

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The algorithm has $\frac{1}{4}$ approximation guarantee and sells at most sk copies of each item, where $s = \log 4\mu km + \frac{2}{k}$.

Randomized Rounding on the fly [KV12]

- What if we don't always actually sell the bundle the bidder asks for?

1. Initialize parameter $P_0 = \frac{L}{4km}$, multiset of all items U_0.
2. For each bidder $i \in [n]$:
   - Ask the demand oracle of the bidder to choose a bundle S_i for the available item multiset U_i.
   - With probability q the bidder gets $R_i = S_i$ and $R_i = \emptyset$ otherwise.
   - Update the number of available items $U_{i+1} = U_i \backslash \mathrm{R}_i$ and the prices $P_{i+1}^j = P_i^j \cdot 2^{1/k}$ for each item $j \in R_i$.

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If the inequality $E[v_i(T \cap U_i)] \geq \frac{1}{2} v_i(T)$ holds for any bidder i and any bundle T, then the algorithm gives a $\frac{q}{8}$ approximation guarantee in expectation.

References for Combinatorial Auctions

- AGT Book: Chapter 11
- Combinatorial Auctions with Decreasing Marginal Utilities. Lehmann, Lehmann, Nisan 2001

- An Impossibility Result for Truthful Combinatorial Auctions with Submodular Valuations. Dobzinski 2011
- Approximation Algorithms for Combinatorial Auctions with Complement-Free Bidders. Dobzinski, Nisan, Schapira 2005

- Online Mechanism Design (Randomized Rounding on the Fly). Krysta, Vöcking 2012