

Sorting

Παράλληλοι Αλγόριθμοι

A.Τέντες

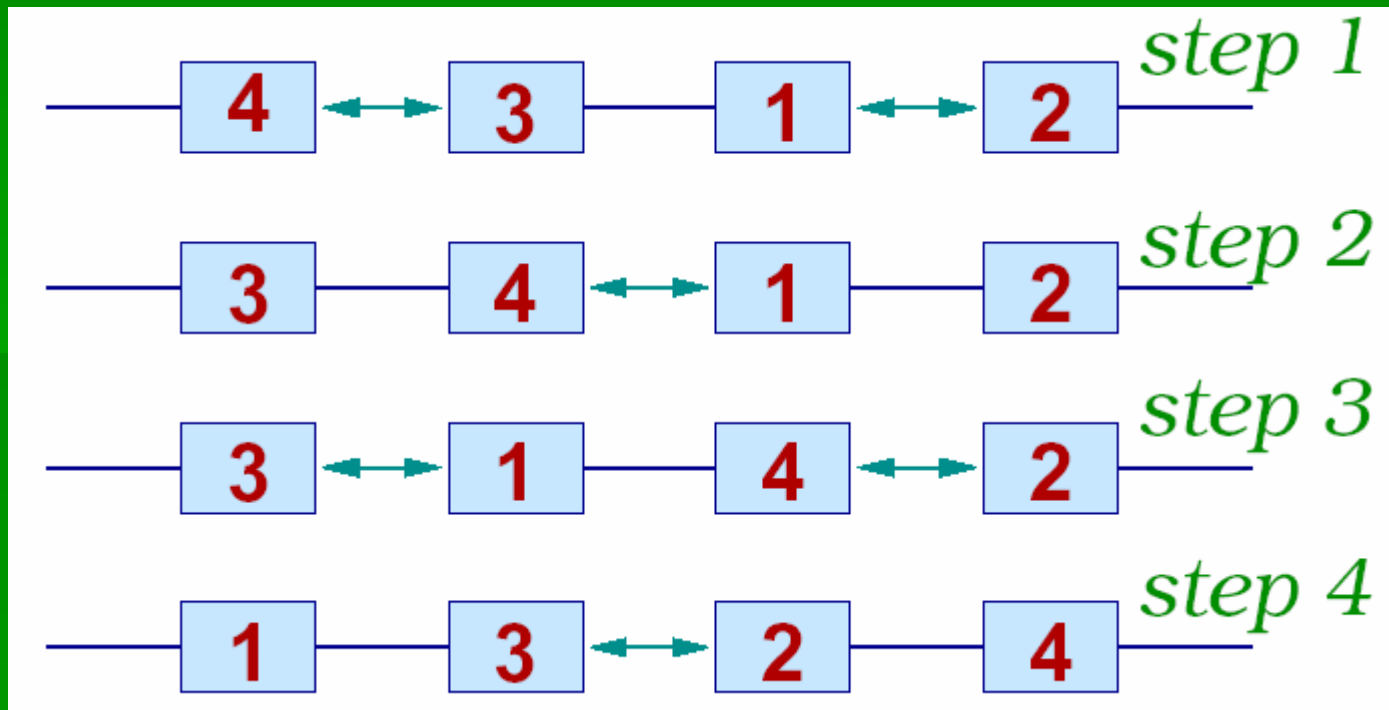
- 0 – 1 Lemma
- Odd-even transposition sort
- Shearsort
- $3n$ sort
- A lower bound

0 -1 Lemma

- If a comparison-exchange algorithm sorts input sets consisting solely of 0's and 1's, then it sorts all input sets of arbitrary values

Odd - even transposition sort

- N numbers can be sorted on an N-cell linear array in $O(N)$ time: the processors alternate operations with their neighbors



Shearsort (1/4)

- Total steps $N(2\log N + 1)$

- Algorithm steps:

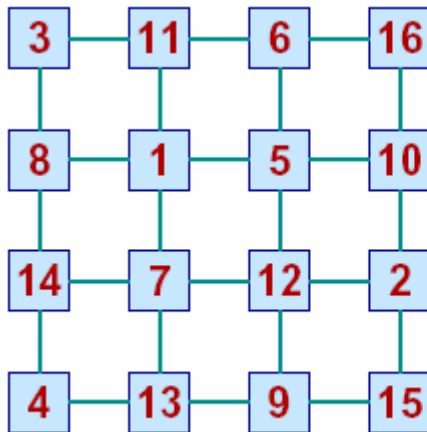
Odd phase: sort each row with odd-even sort (all odd rows are sorted left to right and all even rows are sorted right to left)

Even phase: sort each column with odd-even sort

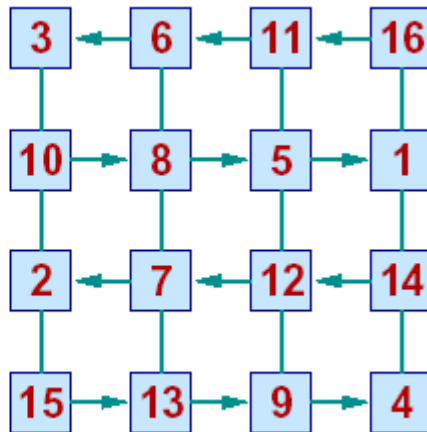
Repeat

Shearsort (2/4)

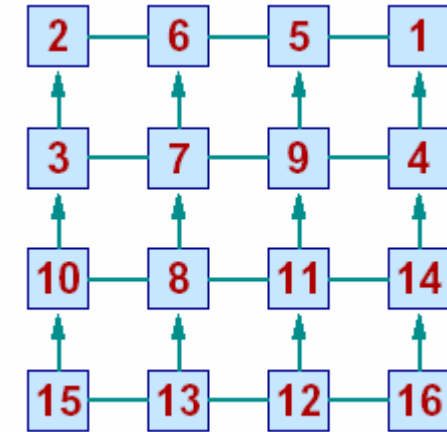
Example:



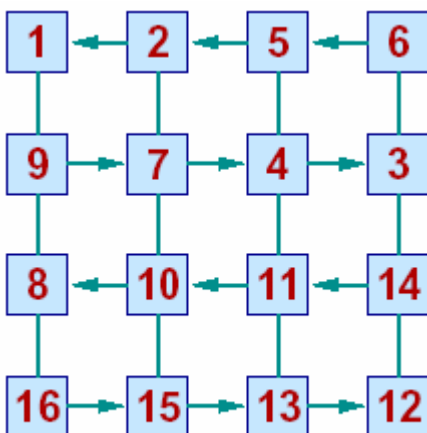
the initial input set



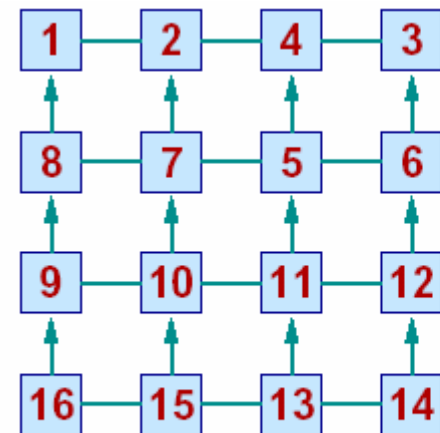
after Phase 1 (row)



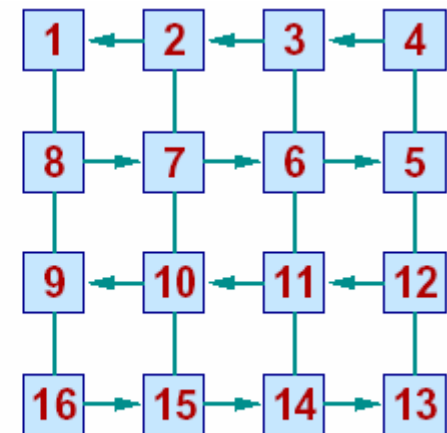
after Phase 2 (column)



after Phase 3 (row)



after Phase 4 (column)

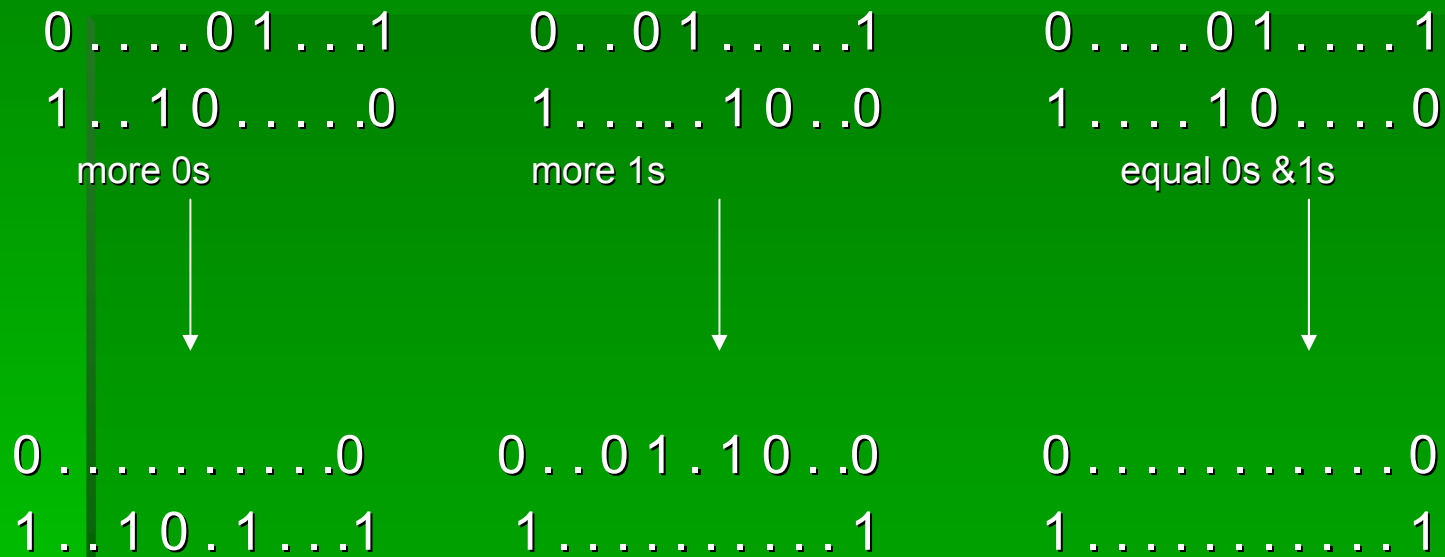


after Phase 5 (row)

Shearsort (3/4)

| | | |
|-------------------|---|---------------|
| 0 0 0 0 0 0 0 0 0 | } | lines with 0s |
| 0 0 0 0 0 0 0 0 0 | | |
| 0 1 1 0 0 1 0 1 0 | } | dirty lines |
| 1 1 1 0 0 1 0 1 0 | | |
| 0 1 0 1 1 0 0 0 1 | | |
| 1 0 1 0 1 1 1 0 1 | | |
| 1 1 1 1 1 1 1 1 1 | } | lines with 1s |
| 1 1 1 1 1 1 1 1 1 | | |
| 1 1 1 1 1 1 1 1 1 | | |

Shearsort (4/4)

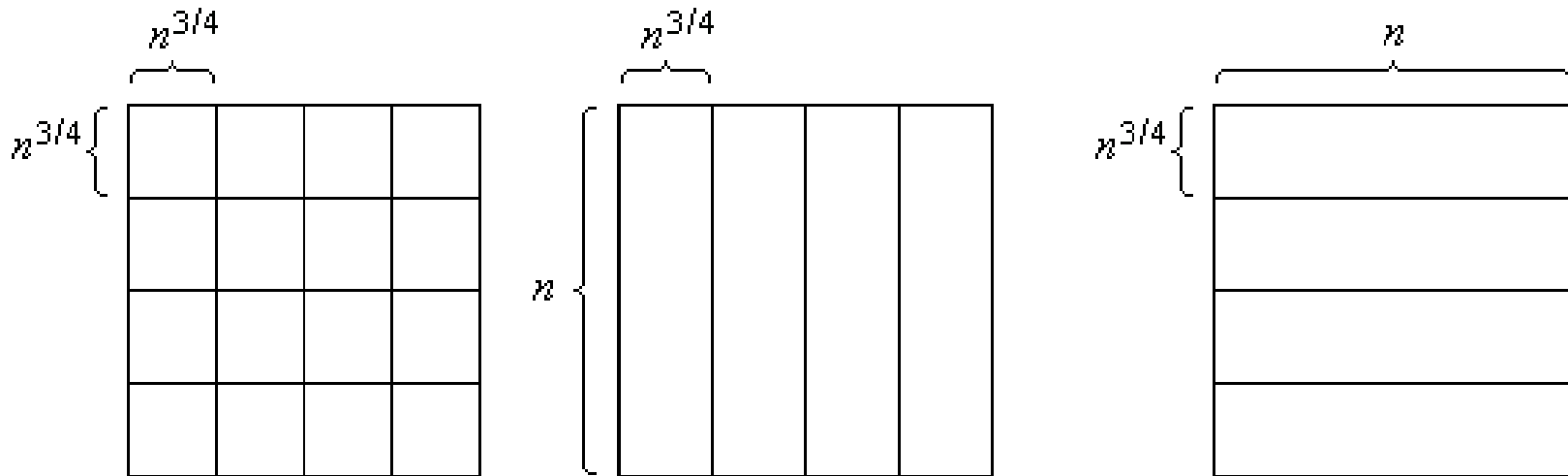


At least half of the rows become 0s or 1s.

Hence after $\log N$ phases the numbers are sorted

3n sort (1/4)


- Schnorr & Shamir '86
- Total steps $3n + O(N^{3/4} \log N)$
- Divide mesh:



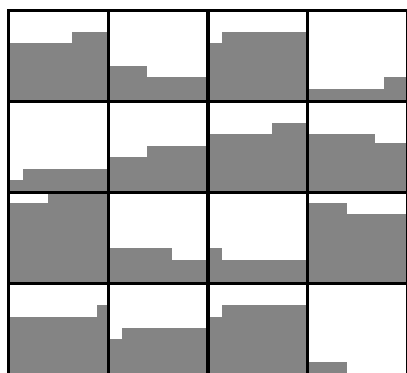
Blocks and slices

3n sort (2/4)

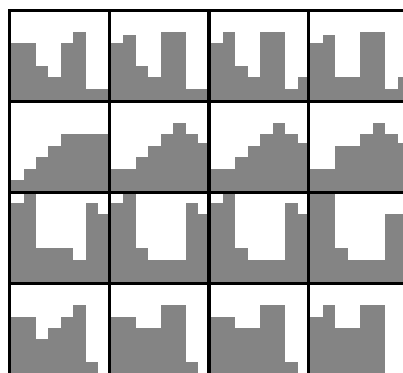
The algorithm phases:

1. Sort the blocks in snakelike order
 2. $N^{1/4}$ -way unshuffle of the columns
 3. Sort the blocks
 4. Sort vertical slices
 5. Sort the columns
 6. Sort each row
 7. Sort the 2 dirty rows using $N^{3/4}$ oets
- Sort the vertical slices
- 

3n sort (3/4)



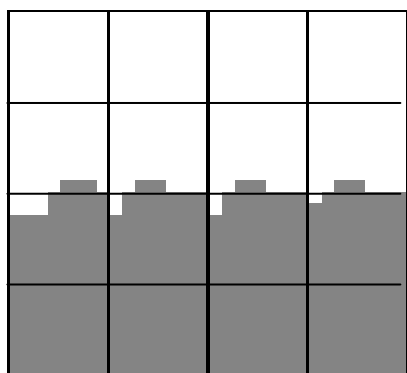
(a) after Step 1



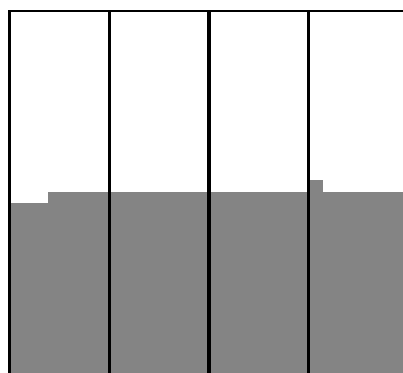
(b) after Step 2



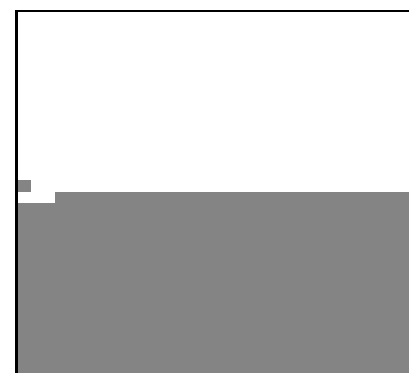
(c) after Step 3



(d) after Step 4



(e) after Step 5



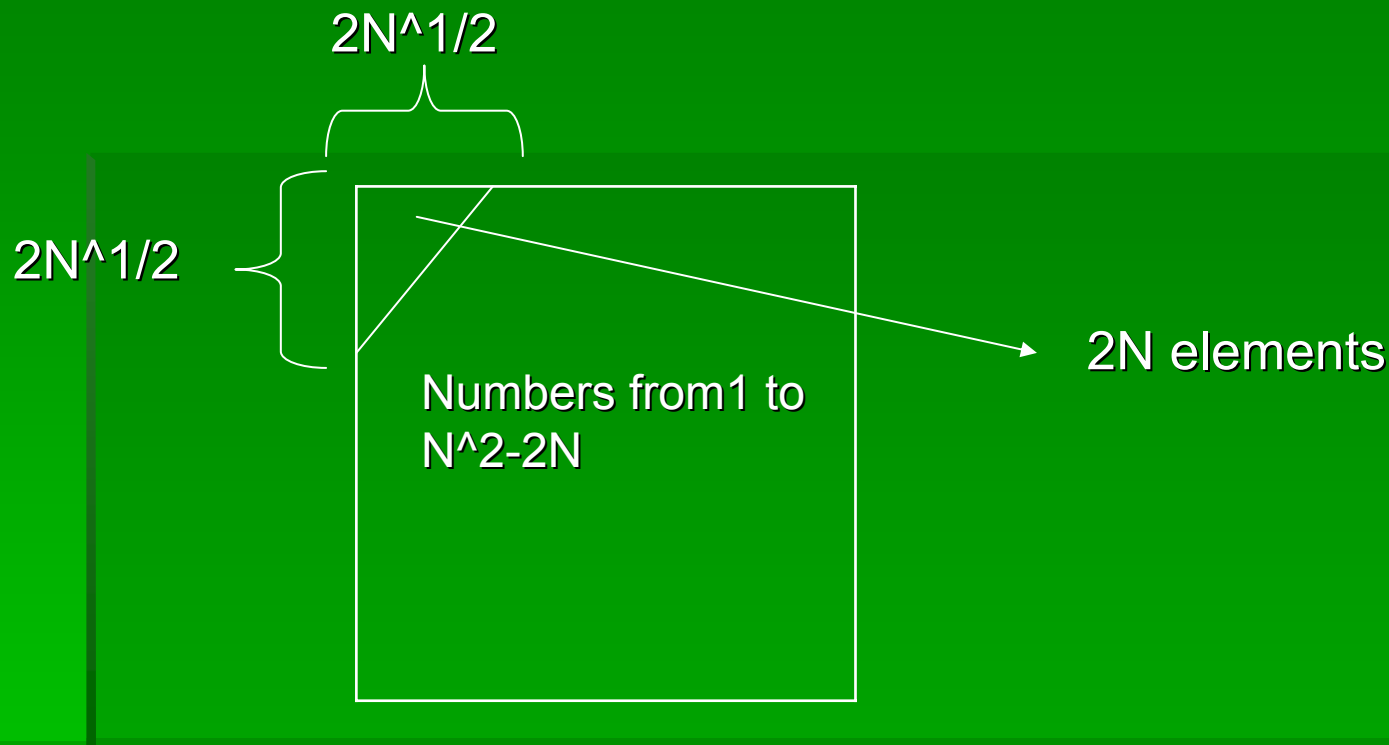
(f) after Step 6

3n sort (4/4)

| | |
|---|---------------------|
| 1: sorting the blocks | $O(n^{3/4} \log n)$ |
| 2: <i>unshuffle</i> along the rows | n |
| 3: sorting the blocks | $O(n^{3/4} \log n)$ |
| 4: sorting the columns | n |
| 5: sorting the vertical slices | $O(n^{3/4} \log n)$ |
| 6: sorting the rows in alternating direction | n |
| 7: $n^{3/4}$ steps of odd-even transposition sort | $O(n^{3/4})$ |

altogether: $3n + O(n^{3/4} \log n)$

A lower bound



- From triangle to (N, N) : $2N - 2N^{1/2} - 1$ steps
- At step $2N - 2N^{1/2} - 2$ in (N, N) no value from triangle
- At least $3N - 2N^{1/2} - 3$ steps needed

Optimal algorithm

- Thomson & Kung '77
- Total steps $3N + O(N^{2/3} \log N)$